CLAIMS:

1. An auction method for holding an auction for a product comprising the steps of:

receiving bids, for each product type in a transaction, that include minimum desired volumes and maximum desired volumes and evaluation prices for said product;

generating a finite set of bids that include as an element said bids that were received; and

employing dynamic programming to generate, using said bid set, a subset of bids wherein the maximum gain is obtained within a range represented by the count of said product available for sale.

- 2. The auction method according to claim 1, wherein said evaluation prices for said product are represented as a non-linear function relative to the desired volume of said product type in said transaction.
- 3. The auction method according to claim 1, further comprising the steps of:

allocating a two-dimensional array V to a memory area by using said dynamic programming;

initializing said two-dimensional array V; and
 recursively solving the recursive equation for said
two-dimensional array V,

wherein

 $V(k,j) := \max \left\{ V(k+1,j), \ V(k,j+1), \ \max_{1_k \le k \le h_k} \{ V(k+1,j+k) + e_k(x) \} \right\}$ is used as the recursive equation, where k denotes an

integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids; s denotes the number of products available for the transaction; e_k denotes the evaluation price when x units of products are purchased according to the bid b_k ; l_k denotes the minimum volume of the bid b_k ; and h_k denotes the maximum volume of the bid b_k .

- 4. The auction method according to claim 3, wherein a bid according to which said product is optimally distributed is selected by back tracking of said two-dimensional array V from the element on the smallest row and in the smallest column.
- 5. The auction method according to claim 1, further comprising:

allocating two-dimensional arrays V and Q to a memory area by using said dynamic programming;

initializing said two-dimensional arrays V and Q; and recursively solving recursive equations for said two-dimensional arrays V and Q,

wherein said evaluation prices for said product represent a linear function relative to the volumes for said product desired for said transaction, and

wherein

$$V(k,j) := \max \left\{ \begin{array}{l} V(k+1,j) \\ V(k,j+1) \\ V(k,j+1) + e_k \\ V(k+1,j+l_k) + e_k l_k \end{array} \right. \\ Q(k,j) := \left\{ \begin{array}{ll} Q(k,j+1) + 1 & (if \ V(k,j) = V(k,j+1) + e_k) \\ l_k & (if \ V(k,j) = V(k+1,j+l_k) + e_k l_k) \\ Q(k,j+1) & (if \ V(k,j) = V(k,j+1) \\ 0 & (otherwise) \end{array} \right. \\ \end{array}$$

is employed as said recursive equation, where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids; s denotes the number of products available for the transaction; e_k denotes the evaluation price in the bid b_k for a product for each unit; l_k denotes the minimum volume of the bid b_k , and h_k denotes the maximum volume of the bid b_k .

- 6. The auction method according to claim 5, wherein a bid according to which said product is optimally distributed is selected by back tracking of said two-dimensional array V from the element on the smallest row and in the smallest column.
- 7. An auction method for performing an auction for multiple products of multiple types comprising the steps of:

receiving bids that each include a combination of said products (including only one type of one product), the volume of said combination desired for the transaction and an evaluation price for said combination;

generating a finite set of bids that include said bids

as elements; and

employing dynamic programming to generate, using said bid set, a subset of bids wherein a maximum gain is obtained within a range represented by the count of said individual products that are available for sale,

wherein said combination of said products satisfies either a first condition $C_i \cap C_j \neq \phi$ or a second condition $C_i \subset C_j$, wherein C_i and C_j (i<j) are two different arbitrary combinations.

- 8. The auction method according to claim 7, wherein the minimum volume and the maximum volume for said combination desired for the transaction are designated as said volumes for said combination.
- 9. The auction method according to claim 7, wherein said evaluation prices for said combination are represented as a non-linear function relative to the volume of said combination desired for the transaction.
- 10. The auction method according to claim 7, further comprising:

a step of sorting said bids in accordance with each combination type by using said dynamic programming method; a step of allocating a two-dimensional array V for said sorted combination C_i ;

an initialization step, which includes the steps of $determining \quad whether \quad a \quad child \quad set \quad of \quad said$ combination $C_{\scriptscriptstyle 1}$ is an empty set and substituting, when the

determination is true, 0 into each element on the $(n+1)\,th$ row of said two-dimensional array V, and

substituting, when the determination is false, the sum of the elements on the first row of said two-dimensional array V, for all the child sets of said combination $C_{\rm i}$, into all the elements on the (n+1)th row of said two-dimensional array V; and

a step of recursively solving said recursive equation of said two-dimensional array V.

Claim 11

The auction method according to claim 10, wherein, as said recursive equation employed is either

 $V(k,j) := \max\{V(k+1,j), V(k,j+1), V(k+1,j+r_k) + r_k e_k\}$

where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids for said combination C_i ; s denotes the minimum number of products available for the transaction, which is included in said combination C_i ; e_k denotes the evaluation price in the bid b_k for one combination; and r_k denotes the volumes of the combinations for the bid b_k , or

 $V(k,\,j) := \max \left\{ V(k+1,\,j), \; V(k,\,j+1), \; \max_{1_k \leq k \leq k_k} \left\{ V(k+1,\,j+x) + e_k(x) \right\} \right\}$

where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids for said combination C_1 ; s denotes the minimum number of products available for the transaction, which is included in said combination C_i ; e_k denotes the evaluation

price when x units of products are purchased according to the bid b_k ; l_k denotes the minimum volume of the bid b_k ; and h_k denotes the maximum volume of the bid b_k , or

$$V(k,j) := \max \left\{ \begin{array}{ll} V(k+1,j) & \\ V(k,j+1) & \\ V(k,j+1) + e_k & (if \ l_k \leq \varrho(k,j+1) \leq h_k) \\ V(k+1,j+l_k) + e_k l_k & \\ \\ \varrho(k,j) := \left\{ \begin{array}{ll} \varrho(k,j+1) + 1 & (if \ V(k,j) = V(k,j+1) + e_k) \\ l_k & (if \ V(k,j) = V(k+1,j+l_k) + e_k l_k) \\ \varrho(k,j+1) & (if \ V(k,j) = V(k,j+1) \\ 0 & (otherwise) \end{array} \right\}$$

where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids for said combination C_i ; s denotes the minimum number of products available for the transaction, which is included in said combination C_i ; e_k denotes the evaluation price in the bid b_k for a product for one combination; l_k denotes the minimum volume of the bid b_k ; and h_k denotes the maximum volume of the bid b_k .

12. The auction method according to claim 11, wherein, for a set C^R that is a subset of the whole set C and has said combination C_i as one element, but whose other elements are not child sets of the other elements of said set C, said two-dimensional array V is tracked backward, beginning with the element on the minimum row and in the minimum column, and after the element on the (n+1)th row is reached, from the first row of a child set of said element on the (n+1)th row, said two-dimensional array V is tracked further

backward to select a bid for the optimal distribution of said products.

13. An auction system for holding an auction for a product comprising:

means for receiving bids, for each product type in a transaction, that include minimum desired volumes and maximum desired volumes and evaluation prices for said product;

means for generating a finite set of bids that include as an element said bids that were received; and

means for employing dynamic programming to generate, using said bid set, a subset of bids wherein the maximum gain is obtained within a range represented by the count of said product available for sale.

- 14. The auction system according to claim 13, wherein said evaluation prices for said product are represented as a non-linear function relative to the desired volume of said product type in said transaction.
- 15. The auction system according to claim 13, further comprising:

means for allocating a two-dimensional array V to a memory area by using said dynamic programming;

means for initializing said two-dimensional array V; and

recursively solving the recursive equation for said two-dimensional array V_{\star}

wherein

 $V(k, j) := \max\{V(k+1, j), V(k, j+1), \max_{l_k \le k \le h_k} \{V(k+1, j+k) + e_k(k)\}\}$ is used as the recursive equation, where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids; s denotes the number of products available for the transaction; e_k denotes the evaluation price when x units of products are purchased according to the bid b_k ; l_k denotes the minimum volume of the bid b_k ; and h_k denotes the maximum volume of the bid b_k .

16. The auction system according to claim 15, further comprising:

means for selecting a bid according to which said product is optimally distributed by back tracking of said two-dimensional array V from the element on the smallest row and in the smallest column.

17. The auction system according to claim 13, further comprising:

means for allocating two-dimensional arrays V and Q to a memory area by using said dynamic programming;

means for initializing said two-dimensional arrays $\mbox{\ensuremath{\text{V}}}$ and $\mbox{\ensuremath{\text{Q}}}\mbox{\ensuremath{\text{z}}}$ and

means for recursively solving recursive equations for said two-dimensional arrays V and $\mathbf{Q}_{\pmb{r}}$

wherein said evaluation prices for said product represent a linear function relative to the volumes for said

product desired for said transaction, and
 wherein

$$V(k, j) := \max \begin{cases} V(k+1, j) \\ V(k, j+1) \\ V(k, j+1) + e_k \\ V(k+1, j+1_k) + e_k l_k \end{cases} (if \ l_k \le \varrho(k, j+1) < h_k) \\ Q(k, j) := \begin{cases} \varrho(k, j+1) + 1 & (if \ V(k, j) = V(k, j+1) + e_k) \\ l_k & (if \ V(k, j) = V(k+1, j+1_k) + e_k l_k) \\ \varrho(k, j+1) & (if \ V(k, j) = V(k, j+1) \\ 0 & (otherwise) \end{cases}$$

is employed as said recursive equation, where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids; s denotes the number of products available for the transaction; e_k denotes the evaluation price in the bid b_k for a product for each unit; l_k denotes the minimum volume of the bid b_k ; and h_k denotes the maximum volume of the bid b_k .

- 18. The auction system according to claim 17, wherein a bid according to which said product is optimally distributed is selected by back tracking of said two-dimensional array V from the element on the smallest row and in the smallest column.
- 19. An auction system for performing an auction for multiple products of multiple types comprising:

means for receiving bids that each include a combination of said products (including only one type of one product), the volume of said combination desired for the

transaction and an evaluation price for said combination;

means for generating a finite set of bids that include said bids as elements; and

means for employing dynamic programming to generate, using said bid set, a subset of bids wherein a maximum gain is obtained within a range represented by the count of said individual products that are available for sale,

wherein said combination of said products satisfies either a first condition $C_i \cap C_j \neq \emptyset$ or a second condition $C_i \subset C_j$, wherein C_i and C_j (i<j) are two different arbitrary combinations.

- 20. The auction system according to claim 19, wherein the minimum volume and the maximum volume for said combination desired for the transaction are designated as said volumes for said combination.
- 21. The auction system according to claim 19, wherein said evaluation prices for said combination are represented as a non-linear function relative to the volume of said combination desired for the transaction.
- 22. The auction system according to claim 19, further comprising:

means for sorting said bids in accordance with each combination type by using said dynamic programming method;

means for allocating a two-dimensional array V for said sorted combination C_1 ;

initialization means, which includes

means for determining whether a child set of said combination C_i is an empty set and substituting, when the determination is true, 0 into each element on the (n+1)th row of said two-dimensional array V, and

means for substituting, when the determination is false, the sum of the elements on the first row of said two-dimensional array V, for all the child sets of said combination C_i , into all the elements on the (n+1)th row of said two-dimensional array V; and

means for recursively solving said recursive equation of said two-dimensional array V.

23. The auction system according to claim 22, wherein, as said recursive equation employed is either

 $V(k,j) := \max\{V(k+1,j), V(k,j+1), V(k+1,j+r_k) + r_k e_k\}$

where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids for said combination C_i ; s denotes the minimum number of products available for the transaction, which is included in said combination C_i ; e_k denotes the evaluation price in the bid b_k for one combination; and r_k denotes the volumes of the combinations for the bid b_k , or

 $V(k, j) := \max \left\{ V(k+1, j), \ V(k, j+1), \ \max_{1_k \leq x \leq h_k} \{ V(k+1, j+x) + e_k(x) \} \right\}$

where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids for said combination C_i ; s denotes the minimum number of products available for the transaction, which is

included in said combination C_i ; e_k denotes the evaluation price when x units of combinations are purchased according to the bid b_k ; l_k denotes the minimum volume of the bid b_k ; and h_k denotes the maximum volume of the bid b_k , or

$$V(k, j) := \max \left\{ \begin{array}{l} V(k+1, j) \\ V(k, j+1) \\ V(k, j+1) + e_k \\ V(k+1, j+1_k) + e_k I_k \end{array} \right. \quad (if \ I_k \leq \varrho(k, j+1) < h_k) \\ \varrho(k, j) := \left\{ \begin{array}{ll} \varrho(k, j+1) + 1 & (if \ V(k, j) = V(k, j+1) + e_k) \\ I_k & (if \ V(k, j) = V(k+1, j+1_k) + e_k I_k) \\ \varrho(k, j+1) & (if \ V(k, j) = V(k, j+1) \\ 0 & (otherwise) \end{array} \right\}$$

where k denotes an integer equal to or greater than 1 and equal to or smaller than n; j denotes an integer equal to or greater than 0 and equal to or smaller than s; n denotes the number of bids for said combination C_i ; s denotes the minimum number of products available for the transaction, which is included in said combination C_i ; e_k denotes the evaluation price in the bid b_k for a product for one combination; l_k denotes the minimum volume of the bid b_k ; and h_k denotes the maximum volume of the bid b_k .

24. The auction system according to claim 23, wherein, for a set C^R that is a subset of the whole set C and has said combination C_i as one element, but whose other elements are not child sets of the other elements of said set C, said two-dimensional array V is tracked backward, beginning with the element on the minimum row and in the minimum column, and after the element on the (n+1)th row is reached, from the first row of a child set of said element on the (n+1)th

row, said two-dimensional array is tracked further backward to select a bid for the optimal distribution of said products.

- 25. A computer-readable storage medium on which a program for holding an auction for a product is stored, said program permitting a computer to perform:
- a function for receiving bids, for each product type in a transaction, that include minimum desired volumes and maximum desired volumes and evaluation prices for said product;
- a function for generating a finite set of bids that include as an element said bids that were received; and
- a function for employing dynamic programming to generate, using said bid set, a subset of bids wherein the maximum gain is obtained within a range represented by the count of said product available for sale.
- 26. A computer-readable storage medium on which a program is stored for performing an auction for multiple products of multiple types, said program permitting a computer to perform:
- a function for receiving bids that each include a combination of said products (including only one type of one product), the volume of said combination desired for the transaction and an evaluation price for said combination;
- a function for generating a finite set of bids that include said bids as elements; and
 - a function for employing dynamic programming to

generate, using said bid set, a subset of bids wherein a maximum gain is obtained within a range represented by the count of said individual products that are available for sale,

wherein said combination of said products satisfies either a first condition $C_i \cap C_j \neq \phi$ or a second condition $C_i \subset C_j$, wherein C_i and C_j (i<j) are two different arbitrary combinations.

27. An auction method for holding an auction for a product comprising the steps of:

receiving bids, for each product type in a transaction, that include a condition concerning said product;

generating a finite set of bids that include as an element said bids that were received; and

employing dynamic programming to generate, using said bid set, a subset of bids wherein the maximum gain is obtained within a range represented by the count of said product available for sale.